

SANT GADGE BABA AMRVATI UNIVERSITY, AMRAVATI
Summer Examination 2020
HVPM's College of Engineering and Technology, Amravati
Department of First Year Engineering
Bachelor of Engineering Sem. :- I & II

Subject :-Engineering Mathematics II

Code :- IBI

Instructions:-

- 1) Solve any two questions**
- 2) All question carry equal marks**

Q1.

a) By using method of partitioning find inverse of A where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

02 Credit Point

b) Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2$ $0 < x < 2\pi$ **02 Credit Point**

c) Show that the reduction formula for $I_{m,n} = \int \cos^m x \cos nx \, dx$ is
 $(m+n)I_{m,n} = \cos^m x \cos nx + m I_{m-1,n-1}$

02 Credit Point

d) Verify the rule of DUIS for the integral $\int_a^{a^2} \log ax \, dx$

01 Credit Point

e) Evaluate $\iint \frac{dx \, dy}{x^4 + y^2}$ where $x \geq 1, y \geq x^2$

01 Credit Point

f) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} \, dz \, dy \, dx$

02 Credit Point

Q2.

a) Find all the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

02 Credit Point

b) Find the Fourier series of $f(x) = 0, -2 < x < -1$

$$k, -1 < x < 1$$

$$0, 1 < x < 2$$

02 Credit Point

c) If $I_n = \int_0^{\pi/4} \sin^{2n} \theta \, d\theta$ then prove that $I_n = \left(1 - \frac{1}{2n}\right) I_{n-2} - \frac{1}{n2^{n+1}}$ **01 Credit Point**

d) Trace the curve $r = a(1 - \cos \theta)$

01 Credit Point

e) Evaluate $\iint (x+y)^2 \, dx \, dy$ over the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

02 Credit Point

f) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$

02 Credit Point

Q3.

a) Determine the rank of A ,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

02 Credit Point

b) If $f(x) = mx, 0 \leq x \leq \frac{\pi}{2}$

$$= m(\pi - x), \frac{\pi}{2} \leq x \leq \pi$$

show that $f(x) = \frac{4m}{\pi} \left(\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} \right)$

02 Credit Point

c) If $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$ then prove that $I_n + I_{n-2} = \frac{1}{n-1}$

01 Credit Point

d) Prove that $\int_0^1 \frac{x^a - x^b}{\log x} dx = \log \left(\frac{a+1}{b+1} \right)$ by DUIS

01 Credit Point

e) change the order of integration and hence evaluate $\int_0^a \int_0^{2\sqrt{xa}} x^2 dx dy$

02 Credit Point

f) Evaluate $\int \int \int \frac{1}{x^2+y^2+z^2} dx dy dz$ throughout the volume of sphere $x^2 + y^2 + z^2 = a^2$

02 Credit Point

Q4.

a) Determine for what values of λ and μ the system of equations:

$$2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$$

have (i) No solution (ii) unique solution (iii) An infinite no of solution.

02 Credit Point

b) The following table gives the variation of periodic current over a period

0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
1.98	1.30	1.05	1.30	- 0.88	- 0.25	1.98

02 Credit Point

c) Show that $B(m, m).B \left(m + \frac{1}{2}, m + \frac{1}{2} \right) = \frac{\pi}{m} 2^{1-4m}$

02 Credit Point

d) Find the length of the arc of the parabola $y^2 = 4ax$ cut of by the line $3y = 8x$

01 Credit Point

e) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dx dy$ by changing into polar coordinates.

02 Credit Point

f) Show by triple integration the volume of sphere of radius a is $\frac{4\pi a^3}{3}$

01 Credit Point