SANT GADGE BABA AMRVATI UNIVERSITY, AMRAVATI
Summer Examination 2020
HVPM's College of Engineering and Technology, Amravati
Department of First year Engineering
Bachelor of Engineering Sem. :- I & II

Subject :-Engineering Mathematics-I **Code :- IAI**

Instructions:-

1) **Solve any two questions**
2) **All question carry equal marks**

Q1.

a) Using Taylor's theorem, express
$(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$ in power of x **01 Credit Point**

b) a] If $u = cosec^{-1}\left(\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right)^{\frac{1}{2}}$

show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{tanu}{144}(13 + tan^2 u)$ **02 Credit Point**

c) If $u = e^x(xcosy - ysiny)$, $v = e^x(xsiny + ycosy)$ and $x = l\xi + m\eta$
$y = l\eta - m\xi$ find $\frac{\partial(u,v)}{\partial(\xi,\eta)}$ **02 Credit Point**

d) Solve the equation $x^9 - x^5 + x^4 - 1 = 0$ using Demoiver's theorem **02 Credit Point**

e) Solve differential equations
$Cos\, x\, \frac{dy}{dx} + y + Sin\, x = 1$ **02 Credit Point**

f) Evaluate.. y = (1 + p)x $+e^p$ **01 Credit Point**

Q2.

a) Evaluate $\lim_{x \to a} \frac{x^a - a^x}{x^x - a^a}$ **01 Credit Point**

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ Show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ **02 Credit Point**

c) If $u = \frac{x+y}{x-y}$, $v = \frac{xy}{(x-y)^2}$ find whether u and v are functionally dependent. If so find the relation between them. **02 Credit Point**

d) If $2cos\frac{\pi}{2^r} = x_r + \frac{1}{x_r}$ prove that $x_1 . x_2 \dots \dots x_\infty = -1$ **02 Credit Point**

e) Solve differential equations

f) $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$ **01 Credit Point**

g) Find the orthogonal trajectories of the family of cardiodes $r = a(1 + cos\,\theta)$
02 Credit Point

Q3.

a) Find n^{th} derivative of $\frac{1}{x^2 - 4x + 3}$ **01 Credit Point**

b) Find $\frac{dy}{dx,}$ if $(cosx)^y = (siny)^x$ **02 Credit Point**

c) The temp. T at any point P(x,y,z) in space is $T=400xyz^2$. Find the highest temp. on the surface $x^2 + y^2 + z^2 = 1$ using Lagrange's method. **02 Credit Point**

d) If $x+\frac{1}{x}=2cos\theta,\ y+\frac{1}{y}=2cos\emptyset$ prove that one of the value of

i] $\frac{x^m}{y^n}+\frac{y^n}{x^m}$ is $2\cos(m\theta-n\emptyset)$, ii] $x^m y^n+\frac{1}{x^m y^n}$ is $2\cos(m\theta+n\emptyset)$.

02 Credit Point

e) Solve differential equations $\left(1+e^{\frac{x}{y}}\right)dx+\ e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\ dy=0$ **02 Credit Point**

f) Evaluate $p-\frac{1}{p}=\frac{x}{y}-\frac{y}{x}$ **01 Credit Point**

Q4.

a) If $cos^{-1}\left(\frac{y}{b}\right)=\log(\frac{x}{n})^n$ prove that

$$x^2y_{n+2}+(2n+1)xy_{n+1}+2n^2y_n=0$$ **02 Credit Point**

b) Given $z=x^nf_1(^y/_x)+y^nf_2(^x/_y)$ show that

$$x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=n^2u$$ **02 Credit Point**

c) Discuss the stationary values of $x^2\ y^2$ -5x^2 -8xy-5y^2 **02 Credit Point**

d) If $sin\varphi=i.tan\theta$ then prove that $cos\theta+isin\theta=\tan(\frac{\pi}{4}+\frac{\varphi}{2})$ **02 Credit Point**

e)Solve differential equations $x\ dy-y\ dx=(x^2+y^2)(x\ dx+y\ dy\ )$ **01 Credit Point**

f) Prove that the system of confocal and coaxial parabolas $y^2=4a(x+a)$ is self orthogonal. **01 Credit Point**