

**SANT GADGE BABA AMRVATI UNIVERSITY, AMRAVATI**  
**Summer Examination 2020**  
**HVPM's College of Engineering and Technology, Amravati**  
**Department of Mechanical Engineering**  
**Bachelor of Engineering Sem. :- III**

**Subject :- Engineering Mathematics MIII**

**Code :-3ME01**

**Instructions:-**

- 1) Solve any two questions**
- 2) All question carry equal marks**

**Q1.**

a) Solve  $(D^3 - D^2 - D + 1)y = \cosh x \cdot \sin x$ . **02 Credit Point**

b) i] Obtain Laplace transform of  $\int_0^t e^{-2t} t \sin^3 t \, dt$

ii] Obtain Inverse Laplace transform of  $\frac{s+29}{(s+4)(s^2+9)}$  **02 Credit Point**

c) Determine the analytic function  $f(z) = u + iv$  whose real part is  $\cos x \cosh y$

**01 Credit Point**

d) Prove that the vector function  $\vec{A} = (x^2 + xy^2)\mathbf{i} + (y^2 + x^2y)\mathbf{j}$  is irrotational and find corresponding scalar potential  $\Phi$  such that  $\vec{A} = \vec{\nabla} \Phi$

**01 Credit Point**

e) Fit a straight line to following data

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

**02 Credit Point**

f) Find real root of the equation  $x^3 + x - 1 = 0$  by regula falsi method upto four decimal places.

**02 Credit Point**

**Q2.**

a) Solve  $(D^2 + 5D + 6)y = e^{-2x} \cdot \sin 2x$  **02 Credit Point**

b) Solve by Laplace transform,

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x} \sin x$  where  $y(0) = 0$  and  $y'(0) = 1$  **02 Credit Point**

c) Find the Bilinear transformation which sends the points  $1, i, -1$  from z-plane into the points  $i, 0, -i$  of w-plane **01 Credit Point**

d) Find the work done in moving a particle once round the circle  $x^2 + y^2 = 9, z = 0$  under the field of force given by

$F = (2x - y + z)\mathbf{i} + (x + y - z^2)\mathbf{j} + (3x - 2y + 4z)\mathbf{k}$  **01 Credit Point**

e) Solve the following partial differential equations:

i]  $((mz - ny)p + (nx - lz)q = ly - mx)$

ii]  $z^2(p^2 + q^2 + 1) = 1$

**02 Credit Point**

f) Use picards method to solve the differential equation  $\frac{dy}{dx} = x^2 + y^2$ , given  $y(0)$ , find  $y_1, y_2$

**02 Credit Point**

**Q3**

a) Solve  $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^2$

**02 Credit Point**

b) i] Evaluate  $\int_0^{\infty} \frac{\cos at - \cos bt}{t} dt$

ii] Using convolution theorem find Laplace transform of  $\frac{1}{(s^2+1)^3}$

**02 Credit Point**

c)  $f(z)$  is analytic then show that

$$\left( \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} \right) |f(z)|^n = n^2 |f(z)|^{n-2} |f'(z)|^2$$

**01 Credit Point**

d) Evaluate the surface integral  $\iint \vec{F} \cdot \vec{n} ds$  where  $\vec{F} = (x + y^2)i - 2xj + 2yzk$  and  $S$  is the surface of the plane  $2x + y + 2z = 6$  in the first octant.

**02 Credit Point**

e) Solve the following partial differential equations

$$yzp + xzq + 2xy = 0$$

**01 Credit Point**

f) Solve by Runge kutta method  $\frac{dy}{dx} = \frac{y-x}{y+x}$ ,  $y(0) = 1$  Find  $y(0.2)$  by taking  $h=0.2$

**02 Credit Point**

**Q4.**

a) Solve  $(D^2 + 3D + 2)y = x \sin 3x$

**02 Credit Point**

b) Find Laplace transform of the triangular wave function of period  $2c$ ,

$$\text{Defined by } f(t) = t \text{ for } 0 < t < c \\ = (2c - t) \text{ for } 0 < t < 2c$$

**02 Credit Point**

c) If  $f(z) = u + iv$  is analytic then find  $f(z)$  if its imaginary part is

$$e^x (x \cos y - y \sin y) \text{ and show that it is harmonic.}$$

**01 Credit Point**

d) Find the directional derivative of the function  $\theta = xy^2 + yz^2 + zx^2$  along the tangent to the

$$\text{curve } x = t, y = t^2, z = t^3 \text{ at the point } (1, 1, 1)$$

**01 Credit Point**

e) Solve the following partial differential equations

$$i] \left( \frac{\partial z}{\partial y} \right)^2 + \left( \frac{\partial z}{\partial x} \right)^2 = \frac{3a^2}{z^2}$$

**02 Credit Point**

f) Solve by Euler's modified method for  $\frac{dy}{dx} = y + e^x$  given  $y(0)=0$ , find  $y(0.4)$  taking  $h=0.2$

**02 Credit Point**