

SANT GADGE BABA AMRVATI UNIVERSITY, AMRAVATI
Summer Examination 2020
HVPM's College of Engineering and Technology, Amravati
Department of Electronics and Telecommunication Engineering
Bachelor of Engineering Sem. :- III

Subject :- Engineering Mathematics MIII

Code :-3ET1

Instructions:-

- 1) Solve any two questions**
- 2) All question carry equal marks**

Q1.

a) Prove that the vector function $\vec{A} = (x^2 + xy^2)i + (y^2 + x^2y)j$ is irrotational and find corresponding scalar potential Φ such that $\vec{A} = \vec{\nabla} \Phi$ **01 Credit Point**

b) Determine the analytic function $f(z) = u + iv$ whose real part is $\cos x \cosh y$ **02 Credit Point**

c) Find real root of the equation $x^3 + x - 1 = 0$ by regula falsi method upto four decimal places. **02 Credit Point**

d) Solve $(D^3 - D^2 - D + 1)y = \cosh x \cdot \sin x$. **02 Credit Point**

e) Solve the following difference equations $U_{x+2} - 2U_{x+1} + 4U_x = 6$ **01 Credit Point**

f) i] Obtain Laplace transform of $\int_0^t e^{-2t} t \sin^3 t dt$

ii] Obtain Inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$ **02 Credit Point**

Q2. a) Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases} \quad \text{02 Credit Point}$$

b) Find the Bilinear transformation which sends the points $1, i, -1$ from z-plane into the points $i, 0, -i$ of w-plane. **01 Credit Point**

c) Use picards method to solve the differential equation $\frac{dy}{dx} = x^2 + y^2$, given $y(0)$, find y_1, y_2 **02 Credit Point**

d) Solve $(D^2 + 5D + 6)y = e^{-2x} \cdot \sin 2x$ **02 Credit Point**

e) Solve $pq = x^m y^n z^{2l}$ **01 Credit Point**

f) Solve by Laplace transform,

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 5y = e^{-x} \sin x \quad \text{where } y(0) = 0 \text{ and } y'(0) = 1 \quad \text{02 Credit Point}$$

Q3) a) Find the directional derivative of the function $\theta = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point $(1,1,1)$ **01 Credit Point**

b) $f(z)$ is analytic then show that

$$\left(\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2}\right) |f(z)|^n = n^2 |f(z)|^{n-2} |f'(z)|^2 \quad \text{01 Credit Point}$$

c) Solve by Runge kutta method $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ Find $y(0.2)$ by taking $h=0.2$ **02 Credit Point**

d) Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^2$ **02 Credit Point**

e) Solve the following partial differential equations

i] $\left(\frac{\partial z}{\partial y}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^2 = \frac{3a^2}{z^2}$

ii] $yzp + xzq + 2xy = 0$ **02 Credit Point**

f) i] Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t} dt$

ii] Using convolution theorem find Laplace transform of $\frac{1}{(s^2+1)^3}$ **02 Credit Point**

Q4. a) Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} \cos x & \text{for } 0 < x < a \\ 0 & \text{for } x > a \end{cases} \quad \text{02 Credit Point}$$

b) If $f(z) = u + iv$ is analytic then find $f(z)$ if its imaginary part is

$e^x (x \cos y - y \sin y)$ and show that it is harmonic. **01 Credit Point**

c) Solve by Euler's modified method for $\frac{dy}{dx} = y + e^x$ given $y(0)=0$, find $y(0.4)$ taking $h=0.2$ **02 Credit Point**

d) Solve $(D^2 + 3D + 2)y = x \sin 3x$ **02 Credit Point**

e) Solve the difference equation

$$Y_{n+2} - 2 \cos \alpha Y_{n+1} + Y_n = \cos n \alpha \quad \text{01 Credit Point}$$

f) Find Laplace transform of the triangular wave function of period $2c$,

$$\text{Defined by } f(t) = \begin{cases} t & \text{for } 0 < t < c \\ (2c - t) & \text{for } 0 < t < 2c \end{cases} \quad \text{02 Credit Point}$$